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Optimal Advertising Campaign Duration of Successive Generation Product using Diffusion of Information

Remica Aggarwal and Udayan Chanda

Department of Management, BITS Pilani, Pilani Campus

E-mail: remica.aggarwal@pilani.bits-pilani.ac.in, udayanchanda@pilani.bits-pilani.ac.in

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Abstract

In the today's global environment , competition is stiff. Marketers are continuously introducing successive generations of product based on latest technology, either to continue to be the leading brands or due to market forces. Due to the dynamic nature of the market, it becomes essential to integrate technological substitution along with diffusion of new products. Advertising of multiple products or multiple generation of a present product involves selecting appropriate advertising medium, analyzing the target market and appropriate utilization of the available advertising budget. An advertising medium demands a huge proportion of the firms' budget to be spent on advertising and therefore determination of an optimal duration of the advertising campaign becomes extremely important for any marketing manager. For an advanced technology product , advertising at right time become even more important. This study developed a mathematical model to determine *the optimal duration of an advertising campaign for an advanced generation product based on diffusion of information in a social group. The optimal timing depends on diffusion coefficient, population size, ad cost per time unit, unit price etc.. The model is based on the assumption that technological advancements do not essentially imply that existing generation products will be withdrawn from the market immediately.*

Keywords: *Mathematical Programming, Multi-product Advertising , Successive Generations*

INTRODUCTION

Technology advances significantly speed up new product development, resulting in several generations of the same product coexist in the consumer market. The most familiar examples include Apple's iPod and iPhone family of products , Samsung android range of mobile products and Microsoft Windows and Offices lines of products. Such Globalization poses challenges before a marketing manager to design their advertising marketing campaigns in a well planned and controlled manner keeping in mind the upcoming versions of the existing products and technology. It is quite obvious that advertising is not a one day's task. A tremendous amount of effort goes into it whether it involves selecting appropriate advertising medium, analyzing the target market and appropriate utilization of the available advertising budget. An advertising medium demands a huge proportion of the firms' budget to be spent on advertising and therefore determination of an optimal duration of the advertising campaign becomes extremely important for any marketing manager. Researches on advertising campaigns have become an interesting hotbed over the last few decades. It is an important point that how much times an advertising campaign is to be implemented in an optimal frame. In other words, what should be the optimal duration of an advertising campaign? Advertising campaigns designed to promote a product in the potential customer market have several objectives such as awareness about the product, attitude of the customer towards the product and sales. In fact, awareness is the key idea of an advertising campaign. Behavior of an individual towards the product is influenced by available information. It is obvious that the more the awareness level of targeted population, the more the sales. Awareness ability is directly related to the diffusion of product information. Product (goods or services) information is, in this article, defined as information about where there is a product available and then buying that product in a particular region. At this point, diffusion theory plays an important role.

When think about advertising for multiple generations., it is again not easy as being a marketing manager , not only you have to think about present generation of product (i.e. the current product) but also about the newer product to be launched. This concept is quite prevalent in media industry when sequel of a movie is prepared or in education industry as well when e-and paperback versions of the present editions are planned. It again depends on the marketing manager of the firm in the way they want to promote their upcoming product. i.e. whether to go for push marketing and spends exhaustively in the beginning so as to capture a bigger amount of market share for the product or to go for light marketing in the beginning when the previous generation product is still in market, so as to capture the maximum from the available product and then consequently advertise heavily for the next generation of the product. This demands a well planned advertising campaign not only for the present product but also for the upcoming product. Hence optimal duration of the advertising campaign is a necessity as we cannot advertise unlimitedly due to budget constraint.

This study developed a mathematical model to determine the optimal duration of an advertising campaign for successive generation high technological product using the diffusion of information in a social group. The model includes total revenue as a function of time, fixed cash flow of advertising cost and all other fringe costs, which are time independent. The paper is arranged as follows: Section 2 is based on literature review. Section 3 provides the modelling framework whereas Section 4 explains the optimization marketing campaign model for the multiple generations focusing on maximizing the profit. Section 4 concludes the paper and provides directions for future work.

LITERATURE REVEIW

Theory of innovation diffusion relates how a new idea, a new product and/or a new service are accepted among the members of its potential consumer population over time. With the process of innovation is linked the process of diffusion. Once an idea takes the shape of a new product, gets successfully developed, business and marketing activities are planned, test marketing is done, the promotional activities are started to communicate the presence of new product in the market to the potential consumer population and there by initiating the process of adoption.

Researches linking advertising campaign duration with consumer behavior are many. [1] considered consumer behavior model for finite advertising campaign duration to determine the optimal dynamic advertising strategy as an optimal control problem. In fact, advertising can be viewed as an investment. [8] adopted the point of view that advertising is an investment, and proposed a simple formula for calculating the level of media spending which maximizes the return on investment. [4]formulated a deterministic model for diffusion of information through a population in which there is a continuous replacement by immigration. Some researchers have studied the relation between advertising campaigns and the number of consumers [5]. [3] developed a model, which describes the spread of opinions among customers based on Ising spin model as an application of statistical physics to marketing. [7] developed a mathematical model to determine the optimal duration of an advertising campaign based on diffusion of information in a social group.

Literature on innovation diffusion modeling and technological substitution is quite rich [2,9-16] . The models they compare for adoption time are the Bass model (no price effect), generalized Bass model and a proportional hazards model which incorporates a Bass model of the baseline adoption. However researches related to advertising policies and successive generation is quite limited.

MODELING FRAMEWORK

The multi-generational diffusion models available in the literature are based heavily on the assumption that diffusion of innovation is a single-phase process, and, thus, completely ignore the different stages of the adoption process.

Notations used in the modeling framework are as follows:

$\overline{N}_1$: Market size when only first generation product is in the market.

$\overline{N}_2 = \overline{N}_1 + K$: Overall market size when second generation product is introduced (increased by K unit due to inception of advance generation product in the market).

$N_j^{/}$: Potential purchasers for j^{th} generation product at time 't' ($j=1, 2$).

p_j: Innovation coefficient for jth generation product ($j=1, 2$).

α *and* β are the choice coefficients for first and second generation products (where, $\alpha + \beta = 1$).

$N_j(t)$: Cumulative number of adopters of j^{th} generation product at time 't' ($j=1, 2$).

$P(t_j)$: Total profit generated by time 't_j', j = 1, 2

P (t_1, t_2): Joint profit function for generation 1 and generation 2

λ : be the price associated with selling a unit of the product in single generation

λ_j : price associated with selling a unit of the product in jth generation ,j=1,2

Aj: advertising cost /time unit for jth generation.

r_j: discount rate in j^{th} generation , j=1,2

$R_1(t_1)$ be the total revenue generated from first generation by time t_1

$R_2(t_2)$ be the total revenue generated from second generation by time t_2

$Fixed\ Cost^{I}$: fixed cost incurred in first generation

$Fixed\ Cost^{II}$: fixed cost incurred in second generation

Diffusion of Information in Single Generation

The single generation diffusion of information is based on the hypothesis that there exists finite population of prospective buyers who with time increasingly adopt the product.

Under the demand assumptions, the mathematical form of rate adoption at any time 't' can be given as:

$$\xi_1(t) = \frac{f_1(t)}{1 - F_1(t)} = p_1 \quad \text{... (1)}$$

The above model suggest that the probability of a potential adopter will purchase a product at time '*t*' given that no adoption has occurred till time '*t*' is depend on the innovation effect p_1.

Here, $\xi_1(t)$ is the hazard rate that gives the conditional probability of a purchase in a small interval of time $(t, t + \Delta t)$, if the purchase has not occurred till time '*t*'.

$f_1(t)$ is the likelihood of purchase at time '*t*'.

$F_1(t) = \int_0^t f_1(t)d$ is the cumulative likelihood of purchasing the product at time '*t*'.

Equation (1) can be rewritten as

$$f_1(t) = p_1[1 - F_1(t)]; \text{ where } F_1(t) = \frac{N_1(t)}{\bar{N}_1} \quad \text{... (2)}$$

Solving equation (2), with the initial condition $F_1(0) = 0$, we have Number of adoptions by time 't' is

$$N_1(t) = \bar{N}_1(1 - e^{-p_1 t}) \quad \text{... (3)}$$

Diffusion of Information for Two Generations

As soon as, second generation product is introduced in the market, two generations of the technology starts competing in the market. As a result, an individual who is confident of both the products can make a choice. The choice behavior of the consumers is governed by the different marketing mix factors.

Assumptions

- An individual is faced with a finite set of choices from which only one can be chosen.
- Individuals belong to a homogenous population, act rationally, possess perfect information and always select the option that maximizes their net personal utility.
- The choice between two alternatives is independent of any other alternative in the choice set.

Let $t_j = t - \tau_j$ where τ_j the introduction time for the j^{th} generation technology. Also, let $(\bar{N}_2 = \bar{N}_1 + K)$ be the increased size of the target market. Let us assume that a fraction α of $\bar{N}_2$ is motivated to choose the first generation technology, while β is motivated to choose the second generation technology. Furthermore, $\alpha + \beta = 1$. Thus, the potential purchsers for first and second generation technologies can be given as:

$$N_1^{/} = \alpha \bar{N}_2 \quad \text{...(5)}$$

$$N_2^{/} = \beta \bar{N}_2 \quad \text{... (6)}$$

By model assumption, adoption process is influenced by the innovation-effect (mass-media) only. Then the adoption rate of first and second generation products in the time interval $t_2 \geq 0$ (i.e. $t \geq \tau_2$) can be given as:

$$\frac{dN_1(t_1)}{dt_1} = p_1\left(N_1^{/} - N_1(t_1)\right) \quad \text{... (7)}$$

$$\frac{dN_2(t_2)}{dt_2} = p_2\left(N_2^{/} - N_2(t_2)\right) \quad \text{... (8)}$$

On solving the above equations with initial condition $N_1(t_1 = \tau_2) = \vartheta$ and $N_2(t_2 = 0) = 0$ we have

$$N_1(t_1) = N_1^/\left(1 - e^{-(p_1 t_1 + \varphi)}\right),$$

where $\varphi = -\left(ln\left(1 - \frac{\vartheta}{N_1^/}\right) + p_1\tau_2\right)$ ... (9)

$N_2(t_2) = N_2^/\left(1 - e^{-p_2 t_2}\right)$ (for $t \geq \tau_2$) ... (10)

The parameters α and β can be interpreted as:

- When, $\alpha < \beta$, then the advance generation product is substituting the first generation product.
- When, $\alpha > \beta$, then the advance generation product fails to create place for itself and the existing generation product is substituting its market share.
- When, $\alpha = \beta$, it means that both the technology generations created a niche for themselves in the market and the substitution effect is negligible.

DEVELOPMENT OF OPTIMAL ADVERTISING CAMPAIGN DURATION MODEL

Assumptions

1. Let us assume that an advertising campaign for a specific product type (goods or services) to a targeted population (people in a particular region) starts at the time $t = 0$.
2. Also assume that the corporation spends on advertising tools such as local TV, radio, printings etc. at a fixed cash flow ($A/time unit) and
3. The corporation has a total cost (*Fixed Cost*) including total buying cost of all products to be desired to sell and any other costs independent of time.
4. It includes other assumptions like the number of people in the region who got information on the product from the advertisements and decided to buy the product is modeled by the diffusion of information process and
5. There are enough products to supply the demand.

Under these assumptions, an objective function to be maximized can be written as

PV of total profit = Present value of total revenue – Net present value of adv. cost – Fixed Cost

Single Generation

Total revenue by time t is $\mathrm{R}(\mathrm{t}_1) = \lambda \mathrm{N}(\mathrm{t}_1) = \lambda \overline{N_1}\left(1 - e^{-p_1 t_1}\right)$ which gives the present value of total revenue $NPVR(t_1) = R(t_1)e^{-rt_1} = \lambda \overline{N_1}\left(1 - e^{-p_1 t_1}\right)e^{-rt_1}$ by continuous discounting. Since the corporation spends at a fixed cash flow as the advertising cost (which depends on t), the net present value of total advertising cost including advertising cost at time t can be presented by the same continuous discount rate as [6]

$$\int_0^{t_1} Ae^{-rt_1}\mathrm{d}t_1 = \frac{A}{r}\left(1 - e^{-rt_1}\right). \quad \text{... (11)}$$

In the light of above considerations, the objective function becomes,

$$\max \mathrm{P}(t_1) = R(t_1)e^{-rt_1} - \frac{A}{r}\left(1-e^{-rt_1}\right) - Fixed\ Cost$$

By differentiating with respect to t_1 to get the necessity condition,

$$P^{'}(t_1) = R^{'}(t_1)e^{-rt_1} - r\left[R(t_1) + \frac{A}{r}\right]e^{-rt_1}, \quad \text{... (12)}$$

By appropriate regulations, we get t_1 as

$$t_1^{*} = -\frac{1}{p_1}\ln\left(\frac{r\lambda\overline{N_1} + A}{(r+p_1)\lambda\overline{N_1}}\right) \quad \text{... (13)}$$

By checking the sufficiency condition, it is clear that $P^{''}(t_1) < 0$ and so the t_1* value is the point that maximizes the total profit function. This means that the optimal duration of the advertising campaign to maximize the total profit is t_1* units. After this time point, with respect to the discount rate, the total profit decreases by the time.

Optimal timing, in this paper, is a time point that the line of possible interest rate of revenue gained from selling the products and the growth rate curve of the objective function intersect. The innovation coefficient p_1 is estimated from the data of historical similar campaigns by some statistical processes and/or simulation techniques.

For Two Successive Generations

First Generation

$$N_1(t_1) = \overline{N_1}\left(1-e^{-p_1t_1}\right) \quad \text{... (14)}$$

$$N_1(t_1) = N_1^{/}\left(1-e^{-(p_1t_1+\varphi)}\right), \quad \text{... (15)}$$

where $\varphi = -\left(ln\left(1-\frac{\vartheta}{N_1^{/}}\right) + p_1\tau_2\right)$

Second Generation

$$N_2(t_2) = N_2^{/}\left(1-e^{-p_2t_2}\right) \text{ (for } t \geq \tau_2\text{)}$$

Total revenue by time t1 is

$$\mathrm{R}_1(\mathrm{t}_1) = \lambda_1\mathrm{N}(\mathrm{t}_1)$$
$$= \lambda_1\overline{N_1}\left(1-e^{-p_1t_1}\right) + \lambda_1N_1^{'}\left(1-e^{-(p_1t_1+\phi)}\right) \quad \text{... (16)}$$

which gives the present value of total revenue $NPVR_1(t_1) = R_1(t_1)e^{-r_1t_1}$ by continuous discounting. Since the corporation spends a fixed cash flow $Fixed\ Cost^{I}$ as the advertising cost (which depends on t_1), the net present value of total advertising cost including advertising cost at time t_1 can be presented by the same continuous discount rate as [6]

$$\int_0^{t_1} A_1e^{-r_1t_1}\mathrm{d}t_1 = \int_0^{\tau_2} A_1e^{-r_1t_1}\mathrm{d}t_1 + \int_{\tau_2}^{t_1} A_1e^{-r_1t_1}\mathrm{d}t_1 = \frac{A_1}{r_1}\left(1-e^{-r_1t_1}\right) \quad \text{... (17)}$$

In the light of above considerations, the profit function for first generation,

$$\max \mathrm{P}_1(t_1) = R_1(t_1)e^{-r_1t_1} - \frac{A_1}{r_1}\left(1-e^{-r_1t_1}\right) - Fixed\ Cost^{I} \quad \text{... (18)}$$

Similarly the total revenue by time t_2 is $\mathrm{R}_2(\mathrm{t}_2) = \lambda_2\mathrm{N}(\mathrm{t}_2) = \lambda_2N_2^{'}\left(1-e^{-p_2t_2}\right)$ which gives the present value of total revenue $NPVR_2(t_2) = R_2(t_2)e^{-r_2t_2}$ by continuous discounting. Since the corporation spends a fixed cash flow $Fixed\ Cost^{II}$ as the advertising cost (which depends on t_2), the net present value of total advertising cost including advertising cost at time t_2 can be presented by the same continuous discount rate as

$$\int_0^{t_2} A_2 e^{-r_2 t_2} \mathrm{d}t_2 = \frac{A_2}{r_2}\left(1-e^{-r_2 t_2}\right) \quad \text{... (19)}$$

The profit function for second generation

$$\max \mathrm{P}_2(t_2) = R_2(t_2)e^{-r_2 t_2} - \frac{A_2}{r_2}\left(1-e^{-r_2 t_2}\right) - Fixed\ Cost^{II} \quad \text{... (20)}$$

Hence the profit function for a two generation model becomes

$$\begin{aligned}\max \mathrm{P}\ (t_1,t_2) &= \max \mathrm{P}_1(t_1) + \max \mathrm{P}_2(t_2)\\ &= R_1(t_1)e^{-r_1 t_1} - \frac{A_1}{r_1}\left(1-e^{-r_1 t_1}\right) - Fixed\ Cost^{I}\\ &+ R_2(t_2)e^{-r_2 t_2} - \frac{A_2}{r_2}\left(1-e^{-r_2 t_2}\right) - Fixed\ Cost^{II}\end{aligned} \quad \text{... (21)}$$

By partially differentiating with respect to t_1 to get the necessity condition for first generation, $\frac{\partial P(t_1,t_2)}{t_1} = 0$ and taking logarithms on both sides will give

$$t_1^* = -\frac{1}{r_1 + 2p_1 - \tau_2} \ln\left(\frac{r_1\lambda_1\overline{(N_1} + N'_1) + A_1}{((r_1+p_1)\lambda_1\overline{N_1})(r_1+p_1)\lambda_1 N'_1(1-\frac{v}{N'_1})}\right) \quad \text{... (22)}$$

By checking the sufficiency condition, it is clear that $\frac{\partial^2 P(t_1,t_2)}{\partial t_1^{\,2}} < 0$ and so the t1* value is the point that maximizes the total profit function for first generation. This means that the optimal duration of the advertising campaign during first generation to maximize the total profit is t1* units. After this time point, with respect to the discount rate, the total profit decreases by the time.

By partially differentiating with respect to t_2 to get the necessity condition for second generation and taking logarithms on both sides ,

$$\frac{\partial P(t_1,t_2)}{\partial t_2} = 0 \quad \text{... (23)}$$

$$\begin{aligned}&\Rightarrow r_2 t_2 - \ln(r_2\lambda_2\beta\overline{N_2}) - (r_2+p_2)t_2\\ &+ \ln((r_2+p_2)\lambda_2\beta\overline{N_2}) - r_2 t_2 + \ln A_2 = 0\end{aligned} \quad \text{... (24)}$$

gives $t_2^* = -\frac{1}{r_2+p_2}\ln\left(\frac{r_2}{(r_2+p_2)A_2}\right)$ ... (25)

Similarly $\frac{\partial^2 P(t_1,t_2)}{\partial t_2^{\,2}} < 0$ and so the t2* value is the point that maximizes the total profit function for second generation

CONCLUSIONS

For a successive generation product , advertising at the right time is very important and hence determining optimal duration of an advertising campaign so as to maximize the return from all the generations is essential . This study developed a mathematical model to determine the optimal duration of an advertising campaign for a single generation as well as an successive generations of a product based on diffusion of information in a social group. Optimal timing t* is the point that maximizes the total profit for a generation and it depends on diffusion coefficient, population size, ad cost per time unit, unit price etc. The model is based on the assumption that technological advancements do not essentially imply that existing generation products will be withdrawn from the market immediately.

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